# SCMS Seminar

## GUSHEL-MUKAI VARIETIES

**Speaker: Olivier Debarre**

**Université Paris Diderot**

**Lecture**

**Time:** 16:00-17:00, Monday, Nov. 25, 2019
16:00-17:00, Monday, Dec. 02, 2019
16:00-17:00, Monday, Dec. 09, 2019

**Venue:** Room 102, Shanghai Center for Mathematical Sciences

**Abstract:** Gushel-Mukai (or GM for short) varieties are smooth (complex) dimensionally transverse intersections of a cone over the Grassmannian Gr(2, 5) with a linear space and a quadratic hypersurface. They occur in each dimension 1 through 6 and they are Fano in dimensions 3, 4, 5, and 6. The aim of this series of talks is to discuss the geometry, moduli, and Hodge structures of these varieties. It is based on joint work with Alexander Kuznetsov and earlier work of Logachev, Iliev, and Manivel.

These varieties first appeared in the classification of complex Fano threefolds: Gushel showed that any smooth prime Fano threefold of degree 10 is a GM variety. Mukai later extended Gushel's results and proved that all Fano varieties of coindex 3, degree 10, and Picard number 1, all Brill–Noether general polarized K3 surfaces of degree 10, and all Clifford general curves of genus 6 are GM varieties.

Why are people interested in these varieties? One of the reasons why their geometry is so rich is that, to any GM variety is canonically attached a sextic hypersurface in P5, called an Eisenbud–Popescu–Walter (or EPW for short) sextic and a canonical double cover thereof, a hyperk¨ahler fourfold called a double EPW sextic. In some sense, the pair consisting of a GM variety and its double EPW sextic behaves very much (but with more diversity and more

complications, and also some differences) like a cubic hypersurface in P5 and its (hyperk¨ahler) variety of lines. Not many examples of these pairs—a Fano variety and a hyperk¨ahler variety—are known (another example are the hyperplane sections of the Grassmannian Gr(3, 10) and their associated Debarre–Voisin hyperk¨ahler fourfold), so it is worth looking into some detail into one of these.

The first main difference is that there is a positive-dimensional family of GM varieties attached to the same EPW sextic; this family can be precisely described. Another distinguishing feature is that each EPW sextic has a dual EPW sextic (its projective dual). GM varieties associated with isomorphic or dual EPW sextics are all birationally isomorphic.

A common feature with cubic fourfolds is that the middle Hodge structure of a GM variety of even dimension is isomorphic (up to a Tate twist) to the second cohomology of its associated double EPW sextic. Together with the Verbitsky–Torelli theorem for hyperk¨ahler fourfolds, this leads to a complete description of the period map for even-dimensional GM varieties.

In odd dimensions, say 3 or 5, a GM variety has a 10-dimensional intermediate Jacobian and we show that it is isomorphic to the Albanese variety of a surface of general type canonically attached to the EPW sextic.

Another aspect of GM varieties that makes them very close to cubic fourfolds is the rationality problem. Whereas GM varieties of dimensions at most 3 are not rational, and GM varieties of dimensions at least 5 are all rational, the situation for GM fourfolds is still mysterious: some (smooth) rational examples are known but, although one expects, as for cubic fourfolds, that a very general GM fourfold should be irrational, not a single irrational example is known.

Finally, the derived category of GM varieties was studied by Kuznetsov and Perry with this rationality problem in mind. They show that this category contains a special semi-orthogonal component, which is a K3 or Enriques category according to whether the dimension of the variety is even or odd.

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