## TOPICS ON NEVANLINNA THEORY AND COMPLEX HYPERBOLICITIES

### Short Courses and Lectures

(July 14 – July 23, 2019)

### 1. Schedule:

- **July 14:** Morning: 10:30 (Registration for student participants)
- **July 14:** Afternoon: 1:00-3:00 (Ru), 4:00-5:00 (Ru)
- **July 15:** 9:00-11:00 (Ru), 2:00-3:00 (Ru), 4:00-5:00 seminar
- **July 16:** 9:00-10:00 (Cherry), 11:00-12:00 (Ru), 2:00-3:00 (Ru), 3:00-5:00 seminars
- **July 17:** 9:00-12:00 (Cherry), Afternoon: Seminars
- **July 18:** 8:30-10:30 (Deng), 11:00-12:00 Deng, 2:00-3:00 (Noguchi), 4:00-5:00 (Deng)
- **July 19:** 8:30-10:30 (Noguchi), 11:00-12:00 (Deng), 2:00-3:00 (Deng), 4:00-5:00 (Deng)
- **July 20-21:** Free discussion and reserved for seminars
- **July 22:** 8:30-10:30 (Sibony), 11:00-12:00 (Deng), 2:00-3:00 (Deng), 4:00-5:00 (Deng)
- **July 23:** 9:00-11:00 (Sibony), 2:00-5:00 seminars

<table>
<thead>
<tr>
<th></th>
<th>July 14</th>
<th>July 15</th>
<th>July 16</th>
<th>July 17</th>
<th>July 18</th>
<th>July 19</th>
<th>July 20/21</th>
<th>July 22</th>
<th>July 23</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>10:30</td>
<td>9:00-11:00</td>
<td>9:00-10:00</td>
<td>9:00-12:00</td>
<td>8:30-10:30</td>
<td>8:30-10:30</td>
<td>Free discussion and reserved for seminars</td>
<td>8:30-10:30</td>
<td>9:00-11:00</td>
</tr>
<tr>
<td></td>
<td>Registration</td>
<td>Ru</td>
<td>Cherry</td>
<td>Cherry</td>
<td>Deng</td>
<td>Noguchi</td>
<td></td>
<td>Sibony</td>
<td>Sibony</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11:00-12:00</td>
<td>11:00-12:00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Cherry</td>
<td>Deng</td>
<td></td>
<td>Deng</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Lunch break</td>
<td>Lunch break</td>
<td>Lunch break</td>
<td>Lunch break</td>
<td>Lunch break</td>
<td>Lunch break</td>
<td>Lunch break</td>
<td>Lunch break</td>
</tr>
<tr>
<td>Time</td>
<td>1:00-3:00</td>
<td>2:00-3:00</td>
<td>2:00-3:00</td>
<td>2:00-3:00</td>
<td>2:00-3:00</td>
<td>2:00-3:00</td>
<td>2:00-3:00</td>
<td>2:00-3:00</td>
<td>2:00-5:00</td>
</tr>
<tr>
<td></td>
<td>Ru</td>
<td>Ru</td>
<td>Ru</td>
<td>Seminars</td>
<td>Ru</td>
<td>Ru</td>
<td>Noguchi</td>
<td>Deng</td>
<td></td>
</tr>
<tr>
<td>Time</td>
<td>4:00-5:00</td>
<td>4:00-5:00</td>
<td>3:00-5:00</td>
<td>4:00-5:00</td>
<td>4:00-5:00</td>
<td>4:00-5:00</td>
<td>4:00-5:00</td>
<td>4:00-5:00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ru</td>
<td>Seminar</td>
<td>Seminar</td>
<td>Ru</td>
<td>Deng</td>
<td>Deng</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ru: Ringrose
Cherry: Cherry
Deng: Deng
Sibony: Sibony
Noguchi: Noguchi

Lunch break
2. Title and abstract

2.1. William Cherry: The potential theoretic method of Eremenko and Sodin
Abstract: The two traditional approaches to proving Nevanlinna's Second Main Theorem are via the logarithmic derivative lemma or via negative curvature. A 1991 paper of Eremenko and Sodin introduced a third potential-theoretic approach. This approach has a disadvantage in that it cannot detect "ramification" and therefore cannot prove the second main theorem with truncated counting functions. But, the fact that it does not depend on derivatives has certain advantages. First, in the case of holomorphic curves in projective space, the method applies just as easily to non-linear hypersurfaces. Second, the method can be made effective to give, for instance, explicit estimates on the ratio of the Kobayashi metric to the Fubini-Study metric on hyperplane complements. I will introduce the method by explaining the proof that a holomorphic curve omitting $2n+1$ hypersurfaces in general position in $n$-dimensional complex projective space must be constant. I will then discuss how the method can be refined to give effective Landau and Schottky type theorems for maps from the disc. Time permitting, I will explain the method in more detail to illustrate how it can be used to prove defect relations, as in the original paper of Eremenko and Sodin.

2.2. Deng Ya: Algebraic differential equations and hyperbolicity of complex manifold
Abstract: In complex geometry, the Kobayashi metric is a pseudometric intrinsically associated to any complex manifold. A complex manifold $X$ is Kobayashi hyperbolic if its Kobayashi pseudometric is a metric; in particular, every holomorphic map from the complex plane $\mathbb{C}$ to $X$ is constant. By Lang and Vojta, the study of Kobayashi hyperbolicity has some analogy with the study of rational points of variety over number fields, a central topic of number theory. After the work of Green-Griffiths, Siu-Yeung and Demailly, the technique of algebraic differential equations is crucial in studying the hyperbolicity of algebraic varieties, and it is important in the recent progress on the Kobayashi conjecture. In this mini-course, I will start with the classical material on the relations between different notions of hyperbolicity for complex varieties, as well as the important and intriguing conjectures. Then I will present the fundamental tools: jet spaces, jet differentials, Demailly-Semple towers, fundamental vanishing theorems. The rest of the lectures will be devoted to several advanced topics: Bloch's theorem on entire curves on complex torus; Voisin's theorem on algebraic hyperbolicity of very general hypersurfaces in the complex projective spaces of optimal degrees; Demailly's holomorphic Morse inequality and its application to the existence of algebraic differential equations on varieties of general type; Siu's slanted vector fields on jet spaces, and Diverio-Merker-Rousseau's proof of weak hyperbolicity of general hypersurfaces of high degrees. The ultimate goal will be to present the recent work on the Kobayashi conjecture (there are now several different proofs by Siu, Brotbek, Demailly, Riedl-Yang). If time allows, I will briefly discuss some recent progress on hyperbolicity of moduli spaces -- an independent but interesting topic on hyperbolicity.
2.3. Junjiro Noguchi: A new introduction of S.C.V.-the Oka theory
Abstract: We here present an elementary simplified approach to give the complete proofs of the Big Three Problems (Approximation, Cousin, Levi) summarized by Behnke-Thullen that were solved by K. Oka and provide the foundation of Several Complex Variables. We formulate and prove a weak coherence theorem only by power series expansions, which is sufficient to prove Joku-Iko, Approximation, Cousin I/II and Levi Problems for unramified Riemann domains. This new approach enables us to complete the proofs of those problems without Weierstrass' Preparation Theorem or the cohomology theory of Cartan-Serre, nor \( L^2 \)-dbar method of Hoermander.

2.4. Min Ru: Introduction to the Nevanlinna theory
Abstract: In this short course, I'll give an introduction to the Nevanlinna theory. The topics include:

2.5. Nessim Sibony: Nevanlinna's theory, a dynamical point of view
Abstract: The goal is to draw some analogies between holomorphic dynamical systems and equidistribution problems in Nevanlinna's Theory. A sample of the questions considered is as follows.
Consider a holomorphic map \( f \) from the unit disc \( D \) to a compact Kähler manifold \((M, \omega)\). Assume it is of unbounded characteristic. A basic question in Nevanlinna's theory is to study the distribution of preimages under the map \( f \), of subvarieties \( D \) of codimension \( p \) in \( M \); which are in the same cohomology class.
I will consider various versions of this problem, using appropriate averages of preimages, in the sense of currents. The guiding line is that if \( f \) grows fast enough, with respect to an exhaustion of \( D \), then sharp equidistribution results hold. I will emphasis the analogies with similar problems in holomorphic dynamics.